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Planck Scale Unification in a Supersymmetric Standard Model

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Abstract

We show how gauge coupling unification near the Planck scale $M_p \sim 10^{19}$ GeV can be achieved in the framework of supersymmetry, facilitating a full unification of all forces with gravity. Below the conventional GUT scale $M_{GUT} \sim 10^{16}$ GeV physics is described by a Supersymmetric Standard Model whose particle content is that of three complete **27** representations of the gauge group E_6. Above the conventional GUT scale the gauge group corresponds to a left-right symmetric Supersymmetric Pati-Salam model, which may be regarded as a "surrogate SUSY GUT" with all the nice features of $SO(10)$ but without proton decay or doublet-triplet splitting problems. At the TeV scale the extra exotic states may be discovered at the LHC, providing an observable footprint of an underlying E_6 gauge group broken at the Planck scale. Assuming an additional low energy $U(1)_X$ gauge group, identified as a non-trivial combination of diagonal E_6 generators, the μ problem of the MSSM can be resolved.

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1 Introduction

Gauge coupling unification and the cancellation of quadratic divergences are two of the most appealing features of supersymmetric (SUSY) extensions of the standard model (SM) [1]. It is well known that the electroweak and strong gauge couplings extracted from LEP data and extrapolated to high energies using the renormalisation group (RG) evolution do not meet within the SM. However, in the framework of the minimal supersymmetric standard model (MSSM) [2] the couplings converge to a common value at some high energy scale. This allows one to embed SUSY extensions of the SM into Grand Unified Theories (GUTs), leading to SUSY GUTs based on $SU(5)$ or $SO(10)$.

However, despite their obvious attractions, SUSY GUTs face some serious challenges from the experimental limits on proton decay on the one hand, and the theoretical requirement of Higgs doublet-triplet splitting on the other as recently discussed for example in [3]. Furthermore the unification of gauge couplings near a conventional GUT scale $M_{GUT} \sim 10^{16}$ GeV leaves open the question of a full unification of all the forces with gravity, although this may be achieved in the framework of string unification, including high energy threshold effects [4].

It was suggested some time ago that one should consider replacing the SUSY GUT theory by a Pati-Salam gauge group above $M_{GUT} \sim 10^{16}$ GeV, which plays the role of a "surrogate SUSY GUT" [5], since there is no proton decay or doublet-triplet splitting problem in such a theory. In this scheme the gauge couplings meet at $M_{GUT} \sim 10^{16}$ GeV, as in the MSSM, and are then held together up to the Planck scale by a combination of left-right symmetry and carefully selected matter content chosen so that the $SU(4)_{PS}$ gauge group has the same beta function as the $SU(2)_L \times SU(2)_R$ gauge couplings [5]. However such "theoretical tuning" of the $SU(4)_{PS}$ and $SU(2)_L \times SU(2)_R$ beta functions appears to be somewhat contrived.

Recently a so-called Exceptional Supersymmetric Standard Model (ESSM) has been proposed [6, 7], in which the low energy particle content consists of three $\mathbf{27}$ representations of the gauge group E_6, plus in addition a pair of non-Higgs doublets $H', \overline{H}'$ arising from incomplete $\mathbf{27}', \overline{\mathbf{27}}'$ representations. In the ESSM, gauge coupling unification works even better than in the MSSM [8]. Although the ESSM solves the usual μ problem via a singlet coupling to two Higgs doublets, the presence of the non-Higgs doublets $H', \overline{H}'$ introduces a new μ' problem since in this case a singlet coupling generating μ' is not readily achieved [6, 7]. However the only purpose of including the non-Higgs states $H', \overline{H}'$ is to help achieve gauge coupling unification at $M_{GUT} \sim 10^{16}$ GeV. This allows the possibility of removing the non-Higgs states $H', \overline{H}'$ from the spectrum. Of course the question of gauge coupling unification must then be addressed, which is the subject of the present paper.

In this paper we consider a similar model to the ESSM but without the additional non-Higgs doublets $H', \overline{H}'$. Clearly, without the additional non-Higgs doublets $H', \overline{H}'$, the gauge couplings will no longer converge at $M_{GUT} \sim 10^{16}$ GeV, or any other scale, so at first sight this possibility looks unpromising. However we shall show that, if the theory is embedded into a Pati-Salam theory at $M_{GUT} \sim 10^{16}$ GeV, then, remarkably,

this leads to a unification of all forces with gravity close to the Planck scale. In the region between M_{GUT} and M_p there is not a SUSY GUT but a "surrogate SUSY GUT" based on the Pati-Salam gauge group which resolves the proton decay and doublet-triplet splitting problems of SUSY GUTs, with Planck scale unification achieved in a more natural way than in [5].

Unification in supersymmetric models containing one or three **27** representations of the gauge group E_6 has recently been considered in the literature [9]. Assuming an intermediate Pati-Salam gauge group at the scale 10^{15} GeV at which the Standard Model (SM) couplings satisfy $\alpha_1 = \alpha_2$, it was claimed that the resulting Pati-Salam gauge couplings could subsequently meet at a higher scale about 10^{18} GeV [9]. However the condition $\alpha_1 = \alpha_2$ cannot be consistently applied at the Pati-Salam breaking scale. Instead we find the Pati-Salam breaking scale to be about an order of magnitude larger than the crossing point $\alpha_1 = \alpha_2$, close to $M_{GUT} \sim 10^{16}$ GeV, with full unification close to $M_p \sim 10^{19}$. Planck scale unification has also been considered in non-supersymmetric models in [10]. In our analysis we shall naively extrapolate the two-loop RGEs up to M_p, although in reality we expect new physics effects arising from quantum gravity to set in about an order of magnitude below this. For example, although the Planck scale is usually equated with the Planck mass energy scale M_p given by $M_p = \sqrt{\hbar c/G} \approx 1.2 \times 10^{19}$ GeV/c^2 where G is Newton's constant, the scale at which quantum gravity becomes relevant may be considered to be $(8\pi G)^{-1/2} \approx 2.4 \times 10^{18}$ GeV, where the factor of 8π comes from the Einstein field equation $G^{\mu\nu} = 8\pi G T^{\mu\nu}$, which is sometimes referred to as the reduced Planck scale. It is around this energy scale that an effective quantum field theory of gravity is expected to break down and some new physics takes over since effective quantum field theories of gravity contain corrections to the predictions of General Relativity proportional to powers of E^2/M_p^2 where E is the energy scale of interest. A more precise estimate of the energy scale at which new physics associated with quantum gravity takes over, based on unitarity violation, may be found in [11], and we return to this point later.

The layout of the rest of this paper is as follows. In the section 2 we consider the pattern of symmetry breaking assumed in this paper. In section 3 we consider the two loop RG evolution of gauge couplings in this model from low energies, through the Pati-Salam breaking scale at $M_{GUT} \sim 10^{16}$ GeV, assuming various Pati-Salam breaking Higgs sectors, and show that the Pati-Salam gauge couplings converge close to the Planck scale $M_p \sim 10^{19}$ GeV. In section 4 we shall construct an explicit supersymmetric model of the kind we are considering. Finally we conclude the paper in section 5.

2 Pattern of Symmetry Breaking

The two step pattern of gauge group symmetry breaking we analyse in this paper is:

$$E_6 \overset{M_p}{\overbrace{\longrightarrow}} G_{422} \otimes D_{LR} \overset{M_{GUT}}{\overbrace{\longrightarrow}} G_{321} \tag{1}$$

where the gauge groups are defined by:

$$G_{422} \equiv SU(4) \otimes SU(2)_L \otimes SU(2)_R, \quad G_{321} \equiv SU(3)_c \otimes SU(2)_L \otimes U(1)_Y \tag{2}$$

and we have assumed that the first stage of symmetry breaking happens close to the Planck scale and the second stage happens close to the conventional GUT scale. The first stage of symmetry breaking is based on the maximal E_6 subgroup $SO(10) \otimes U(1)_\psi$ and the maximal $SO(10)$ subgroup $G_{422} \otimes D_{LR}$ corresponding to a Pati-Salam symmetry with D_{LR} being a discrete left-right symmetry. [3]

The pattern of symmetry breaking assumed in this paper is different from that commonly assumed in the literature based on the maximal $SO(10)$ subgroup $SU(5) \otimes U(1)_\chi$ [6, 7, 13]. In particular the Pati-Salam subgroup does not contain the Abelian gauge group factor $U(1)_\chi$. The only Abelian gauge group factor involved in this pattern of symmetry breaking is $U(1)_\psi$, and in the present analysis we assume that this is broken at M_p. However, as discussed in section 4, there are good phenomenological motivations, related to the solution to the μ problem, for preserving a low energy $U(1)'$ gauge group, and this would require the $U(1)_\psi$ gauge group to be preserved. In the present paper we do not consider the effect of including the $U(1)_\psi$ gauge group factor in the RG analysis, however we have checked that Planck scale unification would still be possible, so the results presented here would not be much affected by its inclusion.[4]

The first stage of symmetry breaking close to M_p will not be considered in this paper. We only remark that the Planck scale theory may or may not be based on a higher dimensional string theory. Whatever the quantum gravity theory is, it will involve some high energy threshold effects, which will depend on the details of the high energy theory, and which we do not consider in our analysis.

The second stage of symmetry breaking close to M_{GUT} is within the realm of conventional quantum field theory, and requires some sort of Higgs sector, in addition to the assumed matter content of three **27** representations of the gauge group E_6. In order to break the Pati-Salam symmetry G_{422} to G_{321} at M_{GUT} the minimal Higgs sector required are the G_{422} representations $H_R = (4, 1, 2)$ and $\overline{H}_R = (\overline{4}, 1, \overline{2})$ [14]. When these particles obtain VEVs in the right-handed neutrino directions they break the $SU(4) \otimes SU(2)_R$ symmetry to $SU(3)_c \otimes U(1)_Y$ with the desired hypercharge assignments, as discussed later.

Although a Higgs sector consisting of H_R and $\overline{H}_R$ is perfectly adequate for breaking Pati-Salam symmetry, it does not satisfy D_{LR}. We must therefore also consider an extended Higgs sector including their left-right symmetric partners. A minimal left-right symmetric Higgs sector capable of breaking Pati-Salam symmetry consists of the $SO(10)$ Higgs states $\mathbf{16_H}$ and $\overline{\mathbf{16}}_{\mathbf{H}}$. If complete E_6 multiplets are demanded in the entire theory below M_p, then the Pati-Salam breaking Higgs sector at M_{GUT} may be

[3] Under D_{LR} the matter multiplets transform as $q_L \to q_L^c$, and the gauge groups $SU(2)_L$ and $SU(2)_R$ become interchanged [12].

[4] With $U(1)_\psi$ included in the RG analysis for the $\mathbf{27_H} + \overline{\mathbf{27}}_{\mathbf{H}}$ graph (right panel of figure 1) it may be necessary to increase the effective MSSM threshold to 350 GeV to ensure Planck scale unification for the larger experimental values of the strong coupling constant.

assumed to be $\mathbf{27_H}$ and $\overline{\mathbf{27}}_\mathbf{H}$. Therefore in our analysis we shall consider two possible Higgs sectors which contibute to the SUSY beta functions in the region between M_{GUT} and M_p, namely either $\mathbf{16_H} + \overline{\mathbf{16}}_\mathbf{H}$ or $\mathbf{27_H} + \overline{\mathbf{27}}_\mathbf{H}$, where it is understood that only the Pati-Salam gauge group exists in this region, and these Higgs representations must be decomposed under the Pati-Salam gauge group. No such Higgs sectors were included in the analysis in [9].

When H_R and $\overline{H}_R$ (contained in either $\mathbf{16_H} + \overline{\mathbf{16}}_\mathbf{H}$ or $\mathbf{27_H} + \overline{\mathbf{27}}_\mathbf{H}$) develop VEVs in the right-handed neutrino directions they break the $SU(4) \otimes SU(2)_R$ symmetry to $SU(3)_c \otimes U(1)_Y$ with the desired hypercharge assignments. Six of the $SU(4)$ and two of the $SU(2)_R$ fields are then given masses related to the VEV of the Higgs bosons and the gauge bosons associated with the T^{15} and T^3_R generators are rotated by the Higgs bosons to create one heavy gauge boson and the bino gauge boson associated with $U(1)_Y$. In breaking $SU(4) \otimes SU(2)_R$ to $SU(3)_c \otimes U(1)_Y$ the SM hypercharge generator is a combination of the diagonal generator $T^{15} = \sqrt{\frac{3}{2}}\ diag(\frac{1}{6}, \frac{1}{6}, \frac{1}{6}, -\frac{1}{2})$ of $SU(4)$ and the diagonal generator of $SU(2)_R$, $T^3_R = \frac{1}{2}\ diag(1, -1)$. $T^{15} = \sqrt{\frac{3}{2}}(B-L)/2$ where B and L are the baryon and lepton number assignments of each standard model particle. Comparing these diagonal generators to the hypercharge values we must have $Y = T^3_R + (B-L)/2$. Then one finds the following relation between the hypercharge gauge coupling constant g_Y and the $SU(4)$ and $SU(2)_R$ gauge coupling constants g_4 and g_{2R} respectively [15]:

$$\frac{1}{\alpha_Y} = \frac{1}{\alpha_{2R}} + \frac{1}{\frac{3}{2}\alpha_4} \tag{3}$$

where $\alpha_Y \equiv \frac{g_Y^2}{4\pi}$, $\alpha_{2R} \equiv \frac{g_{2R}^2}{4\pi}$ and $\alpha_4 \equiv \frac{g_4^2}{4\pi}$.

Because the Pati-Salam symmetry, and hence the standard model, is assumed to come from an E_6 group, then all the charges and generators should be correctly normalized.[5] In this case the conventional standard model hypercharge assignments must be modified by a factor of $\sqrt{\frac{5}{3}}$. Therefore Eq.3 should be rewritten in terms of the 'GUT' normalized hypercharge $g_1 \equiv \sqrt{\frac{5}{3}}g_Y$:

$$\frac{5}{\alpha_1} = \frac{3}{\alpha_{2R}} + \frac{2}{\alpha_4} \tag{4}$$

where $\alpha_1 \equiv \frac{g_1^2}{4\pi}$. Eq.4 is the boundary condition for the gauge couplings at the Pati-Salam symmetry breaking scale, in this case M_{GUT}. Due to left-right symmetry, at the Pati-Salam symmetry breaking scale we have the additional boundary condition $\alpha_{2L} = \alpha_{2R}$. In [9] it was assumed that at the Pati-Salam symmetry breaking scale $\alpha_1 = \alpha_{2L} = \alpha_{2R}$ which disagrees with Eq.4, since $\alpha_4 \neq \alpha_{2L} = \alpha_{2R}$ at this scale, as discussed in the next section.

[5] We choose to normalize the E_6 generators G^a by $Tr(G^aG^b) = 3\delta^{ab}$. It then follows that the Pati-Salam and standard model operators are conventionally normalized by $Tr(T^aT^b) = \frac{1}{2}\delta^{ab}$.

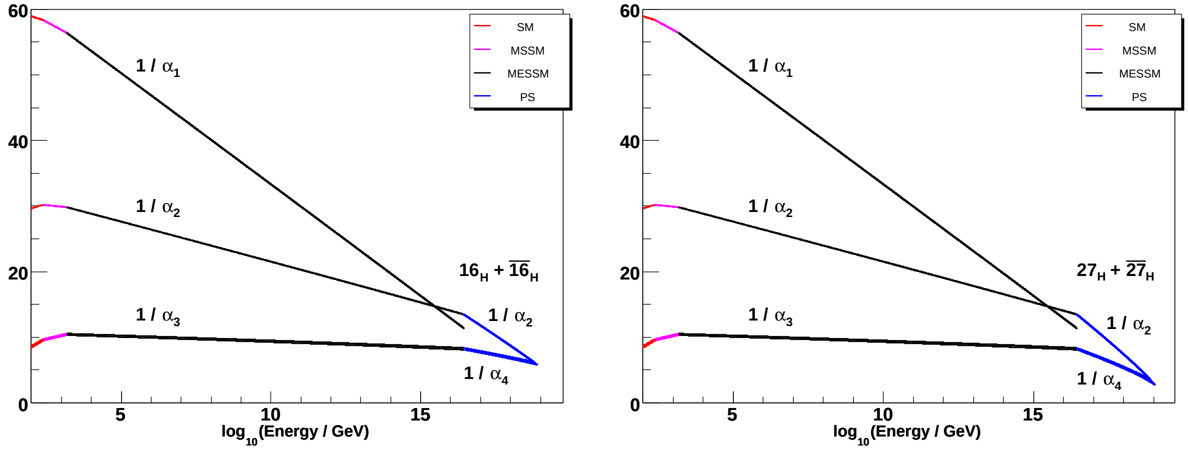

Figure 1: Two-loop Planck Scale Unification in a supersymmetric model which contains three generations of SUSY **27** particles, above an assumed threshold scale of 1.5 TeV. Below this scale the MSSM is assumed with a threshold scale of 250 GeV, below which the SM is assumed. At the scale $M_{GUT} = 10^{16.44(2)}$ GeV the spectrum is embedded into a left-right symmetric Pati-Salam theory, with $\alpha_4 = \alpha_3$ at $M_{GUT} = 10^{16.44(2)}$ GeV, and $\alpha_{2L} = \alpha_{2R}$ above this scale. The left panel contains the additional the SUSY Higgs contained in $\mathbf{16_H} + \overline{\mathbf{16}}_\mathbf{H}$ while the right panel contains the SUSY Higgs in $\mathbf{27_H} + \overline{\mathbf{27}}_\mathbf{H}$ which enter above $M_{GUT} = 10^{16.44(2)}$ GeV. The Pati-Salam gauge couplings α_4 and $\alpha_{2L} = \alpha_{2R}$ converge at $10^{18.83(7)}$ GeV and $10^{18.97(9)}$ GeV for the left and right panels respectively, close to the Planck scale, leading to a unified coupling of $\alpha_P = 0.166(7)$ or $\alpha_P = 0.321(46)$. The numbers in parentheses represent the error resulting from the experimental error in the coupling constants.

3 Two-loop RG Analysis: Planck Scale Unification

In this section we perform a SUSY two-loop RG analysis of the gauge couplings, corresponding to the pattern of symmetry breaking discussed in the previous section. According to our assumptions there are three complete **27** SUSY representations of the gauge group E_6 in the spectrum which survive down to low energies, but, unlike the original ESSM, there are no additional $H', \overline{H}'$ states so the gauge couplings are not expected to converge at M_{GUT}. Instead, we envisage the pattern of symmetry breaking shown in Eq.1, where above the Pati-Salam symmetry breaking scale M_{GUT} we assume, in addition to the three **27** representations, a Pati-Salam symmetry breaking Higgs sector of either $\mathbf{16_H} + \overline{\mathbf{16}}_\mathbf{H}$ or $\mathbf{27_H} + \overline{\mathbf{27}}_\mathbf{H}$ which are assumed to gain masses of order the Pati-Salam symmetry breaking scale M_{GUT}, leaving only the three **27** matter representations below this scale.

For the present RG analysis, we run the couplings up from low energies to high energies, using as input the SM couplings measured on the Z-pole at LEP, which are as follows [16]: $\alpha_1(M_Z) = 0.016947(6)$, $\alpha_2(M_Z) = 0.033813(27)$ and $\alpha_3(M_Z) = 0.1187(20)$. The general two-loop beta functions used to run the gauge couplings can be found in [17]. From M_Z up to an assumed MSSM threshold energy of 250 GeV we consider only the non-SUSY SM spectrum including a top quark threshold at 172 GeV. From 250 GeV to 1.5 TeV we include all the states of the MSSM. From 1.5 TeV up to the Pati-Salam symmetry breaking scale we include all the remaining states which fill out three complete SUSY **27** representations. The assumed threshold energies correspond

to those in [8], where a full discussion of MSSM and ESSM threshold effects is given. The only difference is that here we do not include the $H', \overline{H}'$ states of the ESSM, so the gauge couplings do not converge at M_{GUT}. Instead M_{GUT} is taken to be the Pati-Salam symmetry breaking scale, which is determined as follows.

In the previous section we discussed the relation in Eq.4 between the hypercharge and Pati-Salam coupling constants at the Pati-Salam symmetry breaking scale. This can be turned into a boundary condition involving purely G_{321} couplings constants at the Pati-Salam breaking scale, since $SU(3)_c$ comes from $SU(4)$ so $\alpha_3 = \alpha_4$ at this scale, and, as remarked, D_{LR} symmetry requires that $\alpha_{2R} = \alpha_{2L}$ at the Pati-Salam symmetry breaking scale. Therefore Eq.(4) can be re-expressed as:

$$\frac{5}{\alpha_1} = \frac{3}{\alpha_{2L}} + \frac{2}{\alpha_3}. \tag{5}$$

Having specified the low energy matter content, and thresholds, Eq.5 allows a unique determination of the Pati-Salam breaking scale, by simply running up the gauge couplings until the condition is satisfied. In practice, α_3 runs quite slowly (its one loop beta-function is zero), while the inverse hypercharge coupling decreases most rapidly and the condition is satisfied for a Pati-Salam symmetry breaking scale about an order of magnitude higher energy scale than the crossing point of α_1 and α_2 assumed in [9]. Assuming the above matter content and threshold corrections, the Pati-Salam symmetry is found to be broken at $M_{GUT} = 10^{16.44(4)}$ GeV as illustrated in Figure 1. This is close to the conventional GUT energy scale, and justifies our use of the notation M_{GUT} to denote the Pati-Salam breaking scale.

Above the scale M_{GUT} we run up the two Pati-Salam gauge couplings, namely α_4 and $\alpha_{2L} = \alpha_{2R}$, including, in addition to the three SUSY **27** matter representations, also a Pati-Salam SUSY Higgs breaking sector consisting of either $\mathbf{16_H} + \overline{\mathbf{16}}_\mathbf{H}$ or $\mathbf{27_H} + \overline{\mathbf{27}}_\mathbf{H}$. The Pati-Salam couplings are found to converge at at either $10^{18.83(7)}$ GeV or $10^{18.97(9)}$ GeV, respectively, as shown in Figure 1.[6] These values are close to the Planck scale $M_p = 1.2 \times 10^{19}$ GeV, and suggests a Planck scale unification of all forces with gravity.

The value of the gauge coupling constant at the unification scales $10^{18.83(7)}$ GeV or $10^{18.97(9)}$ GeV is $\alpha_P = 0.166(7)$ or $\alpha_P = 0.321(46)$ for the $\mathbf{16}_H + \overline{\mathbf{16}}_\mathbf{H}$ or $\mathbf{27_H} + \overline{\mathbf{27}}_\mathbf{H}$ particle spectra, respectively. These values of the unified gauge coupling at the Planck scale are much larger than the conventional values of α_{GUT}, and indeed are larger even than $\alpha_3(M_Z)$, however they are still in the perturbative regime. Of course there are expected to be large threshold corrections coming from Planck scale physics which are not included in our analysis. Indeed, we would expect that QFT breaks down as we approach the Planck scale, so that our RG analysis ceases to be valid as we approach the Planck scale, as remarked in the Introduction. The precise energy scale E_{new} at which quantum field theories of gravity are expected to break down and new physics

[6] If we were to drop the D_{LR} symmetry then, with a minimal Pati-Salam Higgs content consisting of just H_R and $\overline{H}_R$, the equation $\frac{5}{\alpha_1} = \frac{3}{\alpha_{2R}} + \frac{2}{\alpha_3}$ at the Pati-Salam scale would predict that the Pati-Salam symmetry is broken at $10^{14.4(1)}$ GeV and that unification would occur at $10^{19.72(15)}$ GeV.

Z_2	R-charges	Incomplete E_6 multiplets
$+$	$\frac{2}{3}$	$\phi_0 = S_3,\ h_3$
$-$	$\frac{2}{3}$	$\phi_i = F_i,\ F_i^c,\ \mathcal{D}_i,\ h_\alpha,\ S_\alpha$
$+$	$\frac{1}{3}$	$\mathbf{16_H}$ or $\mathbf{27_H}$
$+$	$\frac{1}{3}$	$\overline{\mathbf{16}}_\mathbf{H}$ or $\overline{\mathbf{27}}_\mathbf{H}$
$-$	0	Σ
$+$	$\frac{4}{3}$	M

Table 1: The Z_2 and R-charge assignments of the Pati-Salam respecting incomplete E_6 multiplets of the MESSM. The four **27** multiplets are divided into ϕ_0 and ϕ_i where $i = 1\ldots3$. ϕ_0 contains the MSSM Higgs doublets h_3 and a Pati-Salam symmetry singlet S_3 which gives mass to the $\mathcal{D}_i$ and h_α particles in ϕ_i where $\alpha = 1, 2$. ϕ_i contains the quarks and leptons F_i and F_i^c as well as the exotic quarks $\mathcal{D}_i$ and non-Higgs h_α. The $\mathbf{16_H}$ and $\overline{\mathbf{16}}_\mathbf{H}$ or $\mathbf{27_H}$ and $\overline{\mathbf{27}}_\mathbf{H}$ multiplets of $SO(10)$ and E_6 respectively break the Pati-Salam symmetry and are given a GUT scale mass from the E_6 singlet M. Σ is another E_6 singlet that gets a VEV within the energy range 10^{7-11}GeV to sufficiently suppress proton decay and to allow the exotic particles to decay with a rate that avoids cosmological problems.

takes over is discussed in [11] based on estimates of the scale of violation of (tree-level) unitarity. An upper bound for this new physics energy scale is given by $E_{new}^2 = 20[G(\frac{2}{3}N_s + N_f + 4N_V)]^{-1}$ where N_s, N_f and N_V are the number of scalars, fermions and vectors respectively that gravity couples to. Assuming three low-energy **27** multiplets, E_{new} would be equal to $10^{18.6}$ GeV which sets an upper bound for the scale at which our quantum field theory analysis (and with any corrections from effective quantum gravity theories included) can no longer be trusted. We have shown that the gauge coupling constants are predicted to be very close to one another at this scale and that, if extrapolated, unify just below M_p. We have naively extrapolated the RGEs up to M_p, even though new physics associated with quantum gravity must enter an order of magnitude below this. The fact that the two PS couplings are very close to each other at E_{new}, and are on a convergent trajectory must be regarded, at best, as a suggestive hint of a unification of the gauge fields with gravity in this approach.

4 Constructing a Realistic Model

We have shown that a low energy matter and Higgs content corresponding to three **27** multiplets of the E_6, embedded in a left-right Pati-Salam symmetry at the GUT scale, can lead to Planck scale unification. However, assuming superpotential interactions corresponding to the E_6 respecting operator $\mathbf{27}_i\mathbf{27}_j\mathbf{27}_k$ where i, j and k are family indices, leads to trouble with phenomenology due to proton decay mediated by TeV scale colour triplets, and flavour changing neutral currents (FCNCs) due to multiple Higgs-like doublets. The solution proposed in [7,8] is to consider incomplete E_6 supermultiplets at low energies, which allows extra symmetries to appear at low energies that forbid proton decay and suppress FCNCs, while allowing the exotic colour triplets to decay with a lifetime less than about one second, to avoid conflict with nucleosynthesis. The same symmetries cannot be used here since they would not respect the Pati-Salam symmetry,

so we must therefore seek alternative symmetries that are consistent with the Pati-Salam symmetry.

We now propose a realistic supersymmetric model which contains the matter content of three $\mathbf{27}$'s of E_6 at low energy, arising from four incomplete $\mathbf{27}$'s of E_6 at high energy, and is embeddable in Pati-Salam (PS), without leading to conflict with phenomenology or cosmology. We shall call such a model the minimal exceptional supersymmetric standard model (MESSM) to distinguish it from the ESSM. In the MESSM we shall impose a $Z_2 \otimes U(1)_R$ symmetry, under which incomplete multiplets of four E_6 $\mathbf{27}$ states ϕ_0 and ϕ_i, where $i = 1 \ldots 3$, transform as shown in Table 1. As in the ESSM, we assume that two Higgs doublets which transform under the PS symmetry as $h_3 = (1, 2, 2)$ and Higgs singlet $S_3 = (1, 1, 1)$ arise from the ϕ_0 representation, which replace the Higgs in the ϕ_3, leaving the corresponding states $h_\alpha = (1, 2, 2)$ and $S_\alpha = (1, 1, 1)$ in the $\phi_{1,2}$ representations which do not develop VEVs or couple to quarks or leptons, and are "non-Higgs". The remaining matter content of the ϕ_i, consist of usual quarks and leptons which transform under PS as $F_i = (4, 1, 2)$ and $F^c_i = (\bar{4}, 1, 2)$, the Pati-Salam singlets $S_\alpha = (1, 1, 1)$ and coloured vector-like quarks $\mathcal{D}_i = D_i + \overline{D}_i = (6, 1, 1)$. Interactions between the non-Higgs and matter particles and exotic quarks and matter particles from the $\phi_i\phi_j\phi_k$ operator would introduce flavour changing neutral currents and rapid proton decay respectively. We suppress these phenomenologically problematic terms by the Z_2 symmetry. However, forbidding all the interactions between the exotic quarks and matter particles could cause serious cosmological problems since this would ensure that the lightest exotic quark is a stable particle whereas cosmology requires the lightest exotic to decay with a lifetime less than about 0.1s to avoid any problems with nucleosynthesis [18]. Therefore the Z_2 symmetry must be broken by the VEV of a new singlet Σ which is sufficiently large to enable the exotic quarks to decay rapidly, but sufficiently small to be consistent with proton decay. This turns out to be possible as we discuss shortly. As well as the three incomplete $\mathbf{27}$ multiplets of E_6, there are additional $\mathbf{16_H} + \overline{\mathbf{16}}_\mathbf{H}$ or $\mathbf{27_H} + \overline{\mathbf{27}}_\mathbf{H}$ Higgs multiplets that break the Pati-Salam symmetry and are assumed to reside at the GUT scale. To give a GUT scale mass to these Higgs multiplets we introduce another E_6 singlet M, which gets a VEV at the GUT scale.

Using Table 1 the only superpotential terms that are allowed are the following: $\phi_0\phi_0\phi_0$ which generates the term $S_3h_3h_3$ which gives an effective μ-term when S_3 gets a VEV; $\phi_0\phi_i\phi_j$ which generates the rest of the MSSM superpotential terms and gives mass to the "non-MSSM" particles $\mathcal{D}_i$, h_α and S_α; $\frac{1}{M_p}\Sigma\phi_i\phi_j\phi_k$ which allows the exotic quarks to decay once Σ gets a VEV at an energy scale as discussed below; $\frac{1}{M_p}\Sigma\phi_i\phi_0\phi_0$ which gives a mass mixing between Higgs and non-Higgs; $\frac{1}{M_p}\phi_i\phi_j\overline{\mathbf{16}}_\mathbf{H}\overline{\mathbf{16}}_\mathbf{H}$ (or $\frac{1}{M_p}\phi_i\phi_j\overline{\mathbf{27}}_\mathbf{H}\overline{\mathbf{27}}_\mathbf{H}$) which generates right-handed neutrino masses;[7] $\frac{1}{M_p}\phi_0\phi_0\overline{\mathbf{16}}_\mathbf{H}\overline{\mathbf{16}}_\mathbf{H}$ (or $\frac{1}{M_p}\phi_0\phi_0\overline{\mathbf{27}}_\mathbf{H}\overline{\mathbf{27}}_\mathbf{H}$) which are harmless; and $M\mathbf{16_H}\overline{\mathbf{16}}_\mathbf{H}$ (or $M\mathbf{27_H}\overline{\mathbf{27}}_\mathbf{H}$) which gives a GUT scale mass to the Higgs multiplets $\mathbf{16_H} + \overline{\mathbf{16}}_\mathbf{H}$ or $\mathbf{27_H} + \overline{\mathbf{27}}_\mathbf{H}$.

[7] The superpotential term $\frac{1}{M_p}\phi_i\phi_j\overline{\mathbf{16}}_\mathbf{H}\overline{\mathbf{16}}_\mathbf{H}$ is meant to represent all the Pati-Salam operators that are found once the $SO(10)$ multiplet $\overline{\mathbf{16}}_\mathbf{H}$ is decomposed into its Pati-Salam representations. Similar meanings apply to the superpotential terms $\frac{1}{M_p}\phi_i\phi_j\overline{\mathbf{27}}_\mathbf{H}\overline{\mathbf{27}}_\mathbf{H}$, $\frac{1}{M_p}\phi_0\phi_0\overline{\mathbf{16}}_\mathbf{H}\overline{\mathbf{16}}_\mathbf{H}$, $\frac{1}{M_p}\phi_0\phi_0\overline{\mathbf{27}}_\mathbf{H}\overline{\mathbf{27}}_\mathbf{H}$, $M\mathbf{16_H}\overline{\mathbf{16}}_\mathbf{H}$ and $M\mathbf{27_H}\overline{\mathbf{27}}_\mathbf{H}$.

The $D_i + \overline{D}_i$ components of the superpotential term $\phi_i\phi_j\phi_k$ cause proton decay through the decay channels $p \to K^+\overline{\nu}$ via $d = 5$ operators (via the $\phi_0\phi_j\phi_k$ term which is responsible for the triplet mass m_D) and $p \to \pi^0 e^+$ via $d = 6$ operators with matrix elements proportional to $1/m_D$ and $1/m_D^2$ respectively. If the exotic quarks get a mass of order $m_D = 1.5$ TeV from $\phi_0\phi_j\phi_k$ then we estimate that the term $\phi_i\phi_j\phi_k$ must be multiplied by an effective Yukawa coupling smaller than about 10^{-8} for the proton's lifetime to be above 1.6×10^{33} years and 5.0×10^{33} years which are the present experimental limits for the $p \to K^+\overline{\nu}$ and $p \to \pi^0 e^+$ decay modes respectively [19]. In the MESSM the $\phi_i\phi_j\phi_k$ terms are forbidden by a Z_2 symmetry but are effectively generated from the non-renormalizable terms $\frac{1}{M_p}\Sigma\phi_i\phi_j\phi_k$ when Σ gets a VEV which must therefore be less than 10^{11} GeV to avoid experimentally observable proton decay. The effective operators $\phi_i\phi_j\phi_k$ from $\frac{1}{M_p}\Sigma\phi_i\phi_j\phi_k$ are also the source of exotic quark decay in this model and, for the exotic quarks to have a lifetime less than 0.1s, we estimate that the $\phi_i\phi_j\phi_k$ operators must be multiplied by an effective Yukawa coupling no less than 10^{-12}, in which case Σ must get a VEV greater than 10^7 GeV. Therefore, for the model to be phenomenologically acceptable, we require that the E_6 singlet Σ should get a VEV between 10^{7-11} GeV.

As well as the Z_2 and $U(1)_R$ symmetries, a $U(1)_\psi$ symmetry from E_6 may also be assumed. In our RG analysis we assumed for simplicity that $U(1)_\psi$ is broken at M_p. However $U(1)_\psi$ may be unbroken at M_p, and is broken instead at M_{GUT} by the Pati-Salam symmetry breaking Higgs H_R and $\overline{H}_R$. These Higgs also break the diagonal generator T_R^3 of $SU(2)_R$ and the $B - L$ generator of $SU(4)$ down to hypercharge Y, with $Y = T_R^3 + (B - L)/2$ taking a zero value for the right-handed neutrino and anti-neutrino components. However this is not the only Abelian generator that is preserved by this Higgs sector. Under $E_6 \to SO(10) \otimes U(1)_\psi$, $\mathbf{27} \to \mathbf{16_{1/2}} + \mathbf{10_{-1}} + \mathbf{1_2}$, and the $U(1)_\psi$ generator may be written as $T_\psi = diag(1/2, -1, 2)$.[8] The right-handed neutrino component of the Higgs sector which develops the VEV will therefore also preserve the generators $T_\psi - (B - L)/2$ and $T_\psi + T_R^3$, in addition to $Y = T_R^3 + (B - L)/2$. It is straightforward to show that precisely one additional Abelian generator orthogonal to $U(1)_Y$ is preserved, namely:

$$X = (T_\psi + T_R^3) - c_{12}^2 Y \tag{6}$$

where $c_{12} = \cos\theta_{12}$ and the mixing angle is given by

$$\tan\theta_{12} = \frac{g_{2R}}{g_{B-L}}, \qquad g_{B-L} = \sqrt{\frac{3}{2}}\, g_4, \tag{7}$$

where the Pati-Salam coupling constants g_{2R} and g_4 are evaluated at M_{GUT}.

The $U(1)_X$ associated with the preserved generator in Eq.6 is an anomaly-free gauge group which plays the same role in solving the μ problem as the $U(1)_N$ of the ESSM, since it allows the coupling Sh_uh_d which generates an effective μ term, while forbidding S_3 and the $\mu h_u h_d$. $U(1)_X$ is broken by the S singlet VEV near the TeV scale, yielding

[8] The respective E_6 normalized generator is $G^{78} = \frac{1}{\sqrt{6}} diag(\frac{1}{2}, -1, 2)$.

a physical Z' which may be observed at the LHC. We emphasize that this Z' is distinct from those usually considered in the literature based on linear combinations of the E_6 subgroups $U(1)_\psi$ and $U(1)_\chi$ since, in the MESSM, $U(1)_\chi$ is necessarily broken at M_p. In particular the Z' of the MESSM based on $U(1)_X$ and that of the ESSM based on $U(1)_N$ will have different physical properties.

5 Summary

In this paper we have proposed and discussed a supersymmetric standard model, valid below the conventional GUT scale $M_{GUT} \sim 10^{16}$ GeV. The particle content consists of three complete SUSY **27** representations of E_6. However the E_6 gauge group is broken at M_p and the low energy gauge group is just that of the SM, supplemented by an additional $U(1)_X$ gauge group. The Higgs doublets which break electroweak symmetry arise from a **27** representation of E_6 as in the ESSM. The model is an example of a low energy Supersymmetric Standard Model (SSM) whose spectrum contains only complete GUT representations. In this respect it is quite unlike any of the conventional SSM's in the literature such as the MSSM, NMSSM or ESSM which all contain low energy Higgs (or non-Higgs) states which do not form complete GUT representations.

We have shown that while gauge coupling at $M_{GUT} \sim 10^{16}$ GeV is lost, the new model suggests a gauge coupling unification near the Planck scale $M_p \sim 10^{19}$ GeV. Although we have naively extrapolated the RGEs up to M_p, we have discussed the fact that in reality there will be new physics effects associated with quantum gravity that will enter about an order of magnitude below this scale, so the idea of Planck scale unification must be considered with caution. Therefore our results can only be regarded as suggestive of Planck scale unification. All we can say is that a supersymmetric model with the low energy matter content of three **27**'s of E_6, embedded in a Pati-Salam gauge group above M_{GUT}, gives rise to a high energy theory with two gauge couplings which are very close to each other and converging at the scale at which quantum gravity effects are expected to set in.

As remarked, above $M_{GUT} \sim 10^{16}$ GeV the model is embedded into a a left-right symmetric Supersymmetric Pati-Salam model, which may be regarded as a "surrogate SUSY GUT" with all the nice features of $SO(10)$ but without proton decay or doublet-triplet splitting problems. The Pati-Salam gauge group is broken at $M_{GUT} \sim 10^{16}$ GeV by a Higgs sector contained in either $\mathbf{16_H} + \overline{\mathbf{16}}_\mathbf{H}$ or $\mathbf{27_H} + \overline{\mathbf{27}}_\mathbf{H}$, leaving only the desired spectrum below this scale. The three right-handed neutrinos and sneutrinos gain masses set by the scale M_{GUT}^2/M_p. At the TeV scale the extra exotic states of the model, which fill out three complete SUSY **27** representations (minus the three right-handed neutrinos and sneutrinos) may be discovered at the LHC, providing an observable footprint of an underlying E_6 gauge group broken at the Planck scale. We have shown that it is possible to construct a realistic supersymmetric model of this kind, which is consistent with phenomenology and cosmology, by using incomplete E_6 multiplets and assuming a symmetry $Z_2 \otimes U(1)_R$, and we called the resulting model the MESSM to distinguish it from the ESSM.

In our unification analysis performed in this paper we assumed for simplicity that the $U(1)_\psi$ gauge group is broken at M_p. However, in order to solve the μ problem, we have shown that it is necessary that the $U(1)_\psi$ gauge group survives down to $M_{GUT} \sim 10^{16}$ GeV, so that below this scale a low energy $U(1)_X$ gauge group emerges as a linear combination of $U(1)_\psi$ and diagonal Pati-Salam generators. Although we have not considered the effect of the $U(1)_\psi$ and $U(1)_X$ gauge groups in the RG analysis, we have checked that Planck scale unification is still possible if they are included. The phenomenology of a low energy Z', corresponding to a $U(1)_X$ which is not a simple linear combination of the E_6 gauge group $U(1)_\psi$ and $U(1)_\chi$, has not so far been considered in the literature and requires a dedicated study [20].

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